1. A market claims the average weight of a package of hamburger in its meat department is one pound, with a standard deviation of 0.18 lb. A manager decides to test \( H_0 : \mu = 1 \) against the two-sided alternative \( H_a : \mu \neq 1 \). It decides to reject \( H_0 \) if a sample mean of 35 packages differs from 1 by more than 1.5 standard deviations. What’s the probability of a Type I error?

2. A market claims the average weight of a package of hamburger in its meat department is one pound, with a standard deviation of 0.18 lb. A manager decides to test \( H_0 : \mu = 1 \) against the one-sided alternative \( H_a : \mu < 1 \) at the \( \alpha = 0.01 \) level of significance. To do this, he randomly selects 35 packages of meat.

   a. For what range of values of \( \bar{x} \) would you reject the null hypothesis?
b. The manager will reject the null hypothesis if $\bar{x} < 0.93$. Suppose the mean weight of all hamburger packages is really 0.9 lb. Find the probability of a Type II error and the power of the test.

3. Suppose we know that the serum cholesterol levels for all 20 to 24-year-old males in the United States is normally distributed with a mean of 180 mg/100ml and the standard deviation is 46 mg/100ml. We would expect that the mean cholesterol level of a special diet group in this population to be higher than 180 mg/100ml. (Assuming that the cholesterol levels are normally distributed and have the same standard deviation, i.e., 46 mg/100ml.) We want to test the mean cholesterol level of this special diet group to be higher than 180 mg/100ml. If we want to risk a 5% (or say, with the power of the test 0.95) chance of failing to reject the null hypothesis in case of that the true mean is as large as 211 mg/100ml.

a. How large a sample do we need?

b. Assuming that we use a sample of size 25 with sample mean of 190, test the hypothesis using P-value approach with a level of significant $\alpha = 0.05$. 
c. Let assume that the alternative hypothesis is $H_a: \mu = 211$ mg/100ml. If $H_a$ is true, what is the probability of accepting $H_0$, i.e. $\beta$, and what would be the power of the test? That is to say “how powerful can this test detect a 31 mg/100ml increase in average cholesterol level?”

4. Your friend bets you $20 that he is better at the video game Pong than you are. You play five games and your friend wins 4 of them. At the significance level 0.10, is this conclusive evidence that your friend is better at Pong than you are?

[We are testing $H_0: p \leq 0.50$ versus $H_a: p > 0.50$, where $p = $ true proportion of times my friend wins in Pong.]

5. Each of a group of 20 intermediate tennis players is given two tennis rackets, one with the two rackets, each player will be asked to state a preference for one of the two types of strings. Let $p$ denote the proportion of all such players who would prefer gut to nylon, and let $X$ be the number of players in the sample who prefer gut. Because gut strings are more expensive, consider the null hypothesis that at most 50% of all such players prefer gut. We simplify this to $H_0: p = 0.5$, planning to reject $H_0$ only if sample evidence strongly favors gut strings i.e. The rejecting regions $\{15, 16, 17, 18, 19, 20\}$

   a. What is the probability of a type I error for the chosen region of part (a)?
b. If 80% of all enthusiasts prefer gut, calculate the probability of a type II error using the appropriate region from part (a)

c. If 13 out of the 20 players prefer gut, should Ho be rejected using a significance level of 0.10?

6. A certain pen has been designed so that true average writing lifetime under controlled conditions (involving the use of a writing machine) is at least 12 hours. A random sample of 18 pens is selected, the writing lifetime of each is determined, and a normal probability plot of the resulting data supports the use of a one-sample t test.

   a. What hypotheses should be tested if the investigator believes a priori that the design specification has been satisfied?

   b. What conclusion is appropriate if the hypotheses of part (a) are tested, $t^* = -2.5$, and $\alpha = 0.05$?
7. A drug manufacturer believes that the manufacturing process is in control if the standard deviation of the dosage in each tablet is at most 0.10 mg. The process will be shut down if there is overwhelming evidence that the process has excessive variation.

Hypothesis

\( H_0: \sigma \leq 0.10 \) (the variation of the process is acceptable)

\( H_a: \sigma > 0.10 \) (the variation of the process is excessive)

For which values of the sample variance (or) sample s.d should the null hypothesis be rejected based on random sample of size \( n = 19 \) with \( \alpha = 0.05 \)?

8. Buyer’s Digest rates thermostats manufactured for home temperature control. In a recent test, 10 thermostats manufactured by ThermoRite were selected and placed in a test room that was maintained at a temperature of 68°F. The temperature readings of the ten thermostats are shown below.

<table>
<thead>
<tr>
<th>Thermostat</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>67.4</td>
<td>67.8</td>
<td>68.2</td>
<td>69.3</td>
<td>69.5</td>
<td>67.0</td>
<td>68.1</td>
<td>68.6</td>
<td>67.9</td>
<td>67.0</td>
</tr>
</tbody>
</table>

Buyer’s Digest gives an “acceptable” rating to a thermo-stat with a temperature variance of 0.5 or less. Conduct a hypothesis test (with \( \alpha = 0.10 \)) to determine whether the ThermoRite thermostat’s temperature variance is “acceptable”.

Hypothesis

\( H_0: \sigma^2 \leq 0.5 \)

\( H_a: \sigma^2 > 0.5 \)
9. The average speed of vehicles on a highway is studied. Suppose John and Mary is assigned to collect data on the speed of vehicles on this highway. After each person has separately observed 10 vehicles, what is the probability that John’s sample mean will exceed Mary’s sample mean by 2 mph? Assume the standard deviation of vehicle speed is known equal to 6 (\(\sigma\)) mph.

10. Perform a test to see if there is statistically significant difference in average BMI between students who did exercise regularly versus those who did not exercise regularly, at 5% level of significance.

<table>
<thead>
<tr>
<th>Weight:</th>
<th>177</th>
<th>180</th>
<th>130</th>
<th>123</th>
<th>195</th>
<th>170</th>
<th>189</th>
<th>175</th>
<th>154</th>
<th>135</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise:</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weight:</th>
<th>163</th>
<th>190</th>
<th>161</th>
<th>138</th>
<th>135</th>
<th>118</th>
<th>130</th>
<th>159</th>
<th>175</th>
<th>118</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise:</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>
11. A group of investigators are studying a treatment that can help reducing LDL Cholesterol level. The following data shows the LDL at the beginning and the end of the observation period from a sample of participants randomly selected from a specific patient population who received the treatment.

<table>
<thead>
<tr>
<th>Subject ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDL at the beginning</td>
<td>186</td>
<td>144</td>
<td>154</td>
<td>174</td>
<td>165</td>
<td>172</td>
<td>158</td>
</tr>
<tr>
<td>LDL at the end</td>
<td>142</td>
<td>113</td>
<td>101</td>
<td>122</td>
<td>129</td>
<td>136</td>
<td>139</td>
</tr>
</tbody>
</table>

a) Perform a t-test to test whether the average reduction in LDL (use LDL at the beginning minus LDL at the end) is greater than 30, at 5% level of significance.

b) Find the 95% confidence interval for estimating the reduction in LDL for the sample population from the treatment.

c) From the past studies, the standard deviation of the reduction in LDL for this population from this treatment is around 10. Find the sample size so that one can have a 90% power to detect a 4 unit’s average reduction in LDL at 5% level of significance for one-sided t-test.
12. The mammogram story shows both the uncertainty of scientific progress and the slipperiness of scientific truth. Uncertainty is very painful. The idea that science is not giving you certitude is very difficult for many people to accept. Some of the strongest evidence is from a study begun in the 1960’s. It found that after 18 years, 153 out of 30,131 women who had mammograms had died of breast cancer, and 196 out of 30,565 women who did not have the test died of breast cancer. Do mammograms reduce death rates? \( H_0: p_1 \geq p_2 \) and \( H_a: p_1 < p_2 \)

13. A study was performed on patients with pituitary adenomas. The standard deviation of the weights of 12 patients with pituitary adenomas was 21.4 kg. A control group of 5 patients without pituitary adenomas had a standard deviation of the weights of 12.4 kg. At 5% level, we wish to know if the weights of the patients with pituitary adenomas are more variable than the weights of the control group.

14. Suppose \( X_1, \ldots, X_7 \sim \text{Nor}(\mu=3; \sigma^2=6) \), \( Y_1, \ldots, Y_7 \sim \text{Nor}(\mu=-3; \sigma^2=2) \), and everything is independent. Let \( S_X^2 \) and \( S_Y^2 \) denote the sample variances of the \( X_i \)'s and \( Y_j \)'s, respectively. Name the distribution (with parameter(s)) of \( \frac{S_X^2}{S_Y^2} \).