MATH 322 Probability & Statistics  
Spring 2010  
Midterm II

1. Suppose now that the weight of a piece of chocolate is a random variable $W$ that is normally distributed with the parameters $\mu$ and $\sigma = 1$. We are given one piece of chocolate to weigh and the result is $w$ grams. To test $H_0 : \mu = 3$ against $H_a : \mu > 3$. Reject $H_0$ in favor of $H_1$ if $w > 5$. What is the probability of committing type I error? [5]

$$\alpha = \text{Type I error}$$

$$= P \{ \text{Reject } H_0 \mid \text{when } H_0 \text{ is true} \}$$

$$= P \{ w > 5 \mid \mu = 3 \}$$

$$= P \{ \frac{w - \mu}{\sigma} > \frac{5 - 3}{1} \} = P \{ Z > 2 \}$$

$$= 1 - \Phi(2)$$

$$= 0.0228$$

2. A factory that produces screws sells its products in packets of 100. A packet is considered defect if more than 10 screws (out of the 100) are defect. To test whether a packet is defective, 5 screws are picked at random and checked. If at most two of the five are defect, we say that the packet is not defective. Take $H_0$: the number of defect screws in a given packet is 10 and $H_a$: the number of defect screws is more than 10.

a. What is the probability for a type I error? [5]

The probability of a type I error is same as the probability of getting 3, 4 or 5 defective screws in a sample of 5 when $p = \frac{1}{10}$

$$= \binom{5}{3} \left( \frac{1}{10} \right)^3 \left( \frac{9}{10} \right)^2 + \binom{5}{4} \left( \frac{1}{10} \right)^4 \left( \frac{9}{10} \right) + \binom{5}{5} \left( \frac{1}{10} \right)^5 \left( \frac{9}{10} \right)^0$$

$$= 0.00856$$

b. Given that the number of defective is 20. What is the probability for a type II error? [5]

Now the probability of getting a defective screw is $\frac{20}{100} = 0.2$.

Type II error is probability of getting 0, 1 or 2 defective screws in a sample of 5 when $p = 0.2$

$$= \binom{5}{0} (0.2)^0 (0.8)^5 + \binom{5}{1} (0.2)^1 (0.8)^4 + \binom{5}{2} (0.2)^2 (0.8)^3$$

$$\approx 0.942$$
3. A random sample of size 100 is to be taken from a normal population having variance $\sigma^2=81$ in order to test the hypothesis $H_0: \mu = 30$ against $H_a: \mu < 30$. The significance level of the test is $\alpha = 0.025$.

a. The rejection region takes the form $\{ \bar{X} < k \}$. Determine the value of $k$. [5]

\[
\alpha = P(\bar{X} < k \text{ when } \mu = 30) = 0.025
\]

\[
\alpha = P\left( \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} < \frac{k - \mu}{\sigma / \sqrt{n}} \right) = 0.025
\]

\[
\Rightarrow k = 30 - 1.96 \left( \frac{9}{\sqrt{100}} \right) = 28.236
\]

b. Compute $\beta(27)$ and power of the test when $\mu = 27$. [5]

\[
\beta = P\left( \bar{X} > 28.236 \text{ when } \mu = 27 \right)
\]

\[
\beta(27) = P\left( \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} > \frac{28.236 - 27}{9 / 10} \right)
\]

\[
= P\left( Z > 1.373 \right)
\]

\[
= 1 - P\left( Z \leq 1.373 \right) = 0.0853
\]

\[
The \ Power \ of \ the \ Test = 1 - \beta = 0.9147
\]

4. For quality control purposes, it is necessary to assure that the average daily yield of chemicals from a particular process is no less than 803 tons. The daily yields (in tons) for the past week were

785  805  790  793  802.

Assume that the daily yields are normally distributed. Test the Hypothesis $H_0: \mu = 803$ against $H_a: \mu < 803$ using the P-value approach. [10]

\[
\bar{X} = 795 \text{ and } S = 8.336666
\]

\[
T = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{795 - 803}{8.336666 / \sqrt{5}} = -2.14577
\]

\[
P(T < -2.14577)
\]

The P-value is between 0.025 and 0.050.
5. The strength of concrete depends, to some extent, on the method used for drying it. Two different drying methods yielded the results shown in the table for independently tested specimens. The measurements are in pounds per square inch (psi)

<table>
<thead>
<tr>
<th>Method I</th>
<th>Method II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_1 ) = 7</td>
<td>( n_2 ) = 10</td>
</tr>
<tr>
<td>( \bar{x}_1 ) = 3250</td>
<td>( \bar{x}_2 ) = 3240</td>
</tr>
<tr>
<td>( S_1 ) = 210</td>
<td>( S_2 ) = 190</td>
</tr>
</tbody>
</table>

Test the appropriate hypothesis by assuming method I and method II have equal variance

\[ H_0 : \mu_1 - \mu_2 = 0 \]
\[ H_a : \mu_1 - \mu_2 \neq 0 \]

Test statistic

\[ T = \frac{\bar{x}_1 - \bar{x}_2}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2) \text{ d.f.} \]

where

\[ S = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{625,600}{15} = 41,040 \]

\[ T = \frac{10}{198.24 \sqrt{\frac{1}{7} + \frac{1}{10}}} = 0.102359 \]

\[ t_{0.05, 15} = 1.753 \]

Fail to reject \( H_0 \).

Find the P-Value. [5]

\[ 2P(T > 0.102359) > 0.20 \]
6. *Figure Perfect*, Inc., is a women's figure salon that specializes in weight reduction programs. Weights for a sample of clients before and after a six-week introductory program are shown below.

<table>
<thead>
<tr>
<th>Client</th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>140</td>
<td>132</td>
</tr>
<tr>
<td>2</td>
<td>160</td>
<td>158</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
<td>195</td>
</tr>
<tr>
<td>4</td>
<td>148</td>
<td>152</td>
</tr>
<tr>
<td>5</td>
<td>190</td>
<td>180</td>
</tr>
<tr>
<td>6</td>
<td>170</td>
<td>164</td>
</tr>
</tbody>
</table>

Using alpha = .05, test to determine whether the introductory program provides a statistically significant weight loss. What is your conclusion? [10]

\[ H_0: M_D = M_{Bf} - M_{Af} = 0 \]
\[ H_a: M_D = M_{Bf} - M_{Af} > 0 \]

\[ \bar{d} = 6.167 \text{ and } S_d = 6.585, n = 6 \]

\[ T = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{6.167}{6.585/\sqrt{6}} = 2.29 \]

\[ t_{2, n-1} = t_{0.05, 5} = 2.015 \]

Reject \( H_0 \) since \( T > t_{2, n-1} \)

\[ 2.29 > 2.015 \]

There is a significant weight loss after the program.
7. Let $P_1$ and $P_2$ be the probabilities of a child getting paralytic polio for the control and treatment group conditions. The objective was to test $H_0 : P_1 - P_2 = 0$ versus $H_a : P_1 - P_2 > 0$. Supposing the true value of $P_1$ is 0.0003, and $P_2 = 0.00015$. Using a level $\alpha = 0.05$ test, what would be reasonable to ask for sample sizes for $\beta = 0.1$ by assuming the sample sizes are equal? [5]

$$n = \left[ \frac{Z_\alpha \sqrt{(P_1+P_2)(q_1+q_2)/2} + Z_\beta \sqrt{P_1q_1 + P_2q_2}}{d^2} \right]^2$$

where $d = P_1 - P_2$

$$= \left[ 1.645 \sqrt{(0.5)(0.00045)(0.99955)} + 1.28 \sqrt{(0.00015)(0.99985)} \right]^2$$

$$(0.0003^2 - 0.00015^2)^2 $$

$$= \left[ \frac{0.0349 + 0.0271}{0.00015} \right]^2 \approx 171,000$$

8. The sample standard deviation of sodium concentration in whole blood (mEq/L) for $m = 20$ marine eels was found to be $s_1 = 40.5$, whereas the sample standard deviation of concentration for $n = 20$ freshwater eels was $s_2 = 32.5$. Assuming normality of the two concentration distributions, test at level $\alpha = 0.10$ to see whether the data suggests any difference between concentration variances for the two types of eels. [10]

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_a : \sigma_1^2 \neq \sigma_2^2$$

Test statistic $f = \frac{s_1^2}{s_2^2} = \frac{(40.5)^2}{(32.5)^2} = 1.553$

The rejection region is $f > f_{\frac{1}{2}, m-1, n-1}$ (or)

$$f < f_{1-\frac{1}{2}, m-1, n-1}$$

$\alpha = 0.10$

$F_{0.05, 19, 19} = 2.16$

Fail to reject $H_0$ at 10% level.