1. Let \( X_1, X_2, \ldots, X_n \) be a random sample of size \( n \) from distribution with mean \( \mu \) and variance \( \sigma^2 \). Show that \( \bar{X}^2 \) is not an unbiased estimate for \( \mu^2 \). [5]

[Hint \( E(Y^2) = V(Y) + [E(Y)]^2 \).]

2. Consider the following results of 10 tosses of a coin:
\( \text{H; T; T; T; T; H; T; H; T; T} \)

a) Estimate the probability of head (H) for this coin.[5]

b. Estimate the standard error of your estimate. [5]

3. Of \( n_1 \) randomly selected male smokers \( X_1 \) smoked filter cigarettes, whereas of \( n_2 \) randomly selected female smokers, \( X_2 \) smoked filter cigarettes. Let \( P_1 \) and \( P_2 \) denote the probability that a randomly selected male and female, smoke filter cigarettes.

a. Show that \( \hat{P}_1 - \hat{P}_2 \) is an unbiased estimator for \( P_1 - P_2 \). [5]

b. What is the estimated standard error of \( \hat{P}_1 - \hat{P}_2 \). [5]
4. Let \( X \) denote the proportion of allotted time that a randomly selected student spends working on a certain aptitude test. Suppose the pdf of \( X \) is

\[
f(x; \theta) = \begin{cases} 
(\theta + 1)x^\theta & 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

where \( \theta > -1 \). A random sample of ten students yields data

\[x_1 = .92, x_2 = .79, x_3 = .90, x_4 = .65, x_5 = .86, x_6 = .47, x_7 = .73, x_8 = .97, x_9 = .94, x_{10} = .77.\]

a. Use the method of moments to obtain an estimator of \( \theta \) and then compute the estimate for this data. [10]

b. Obtain the maximum likelihood estimator of \( \theta \) and then compute the estimate for the given data. [10]
5. Suppose the time allotted for commercials on a primetime TV program is known to have a normal distribution with a standard deviation of 1.5 minutes. A study of 25 showings gave an average commercial time of 11 minutes. Find the 95% confidence interval for the true population mean, \( \mu \). [5]

6. A random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming normal distribution for the number of words typed per minute; find a 95% upper confidence limit for the average number of words typed by all graduates of this school. [5]

7. Due to the decreasing of interest rates, the First Citizens bank received a lot of mortgage applications. A recent sample of 50 mortgage loans resulted in an average of $128,300. Assume a population standard deviation of $15,000. If a next customer called in for a mortgage loan application, find a 95% prediction interval on this customer’s loan amount. [5]

8. You randomly select and weigh 30 samples of an allergy medication. The sample standard deviation is 1.2 milligrams. Assuming the weights are normally distributed, construct 99% confidence intervals for the population variance and standard deviation.
9. A survey estimated that 20% of all Americans aged 16 to 20 drove under the influence of drugs or alcohol. A similar survey is planned for New Zealand. They want a 95% confidence interval to have a margin of error of 0.04.

   (a) Find the necessary sample size if they expect to find results similar to those in the United States.

   (b) Suppose instead they used the conservative formula based on \( \hat{P} = 0.5 \). What is now the required sample size?

10. A tax assessor wants to assess the mean property tax bill for all homeowners in Madison, Wisconsin. A survey ten years ago got a sample mean and standard deviation of $1400 and $1000.

   (a) How many tax records should be sampled for a 95% confidence interval to have a margin of error of $100?

   (b) In reality, the standard deviation is now $1500. Using the sample size you used in (a), would the margin of error for a 95% confidence interval be less than $100, equal to $100, or greater than $100?