1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n$ from a uniform distribution on the interval $(0, \theta)$. 
   a. Obtain the method of moment estimator of $\theta$. [5]

   b. Is the moment estimator for $\theta$ is unbiased? [5]

2. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n$ from the distribution with the probability density function

   
   \[
   f(x; \theta) = \begin{cases} 
   \frac{1}{\theta} \frac{1-x}{\theta} & 0 \leq x \leq 1 \\
   0 & \text{otherwise}
   \end{cases}
   \]

   A random sample yields the following data

   
   
   $x_1 = .92, x_2 = .79, x_3 = .90, x_4 = .65, x_5 = .86, x_6 = .47, x_7 = .73, x_8 = .97, x_9 = .94, x_{10} = .77.$

   Use the method of moments to obtain an estimator of $\theta$ and then compute the estimate for this data. [10]
b. Obtain the maximum likelihood estimator of $\theta$ then compute the estimate using the data given in (a). [10]

3. Suppose that the waiting times at a customer service desk are exponentially distributed with parameter $\lambda$ and that $X_1, X_2, \text{ and } X_3$ are the waiting times of three random customers. Show that the average waiting time $\bar{X} = \frac{X_1 + X_2 + X_3}{3}$ is an unbiased estimator for $1/\lambda$. [5]

4. Suppose that the waiting times of the customers are independent of one another and that the three customers wait 2.5, 4.25, and 5.25 minutes respectively. Find the estimated standard error of the estimate $\bar{X}$. [5]
5. Suppose that the number of drivers who travel between a particular origin and destination during a one hour time period, \( X \), has a Poisson distribution with parameter \( \lambda \). The numbers of drivers are recorded in each hour span for 5 hours, yielding the following:

\[
25 \quad 20 \quad 11 \quad 26 \quad 15
\]

Based on these five observations, find a method of moment estimator for \( \lambda \).

6. A study of the ability of individuals to walk in a straight line reported that accompanying data on cadence (strides per seconds) for a sample of \( n = 20 \) randomly selected healthy men:

\[
0.95 \quad 0.81 \quad 0.93 \quad 0.95 \quad 0.93 \quad 0.86 \quad 1.05 \quad 0.92 \quad 0.85 \quad 0.81 \\
0.92 \quad 0.96 \quad 0.92 \quad 1.00 \quad 0.78 \quad 1.06 \quad 1.06 \quad 0.96 \quad 0.85 \quad 0.92
\]

A descriptive summary of the data from R are as follows.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>StDev</th>
<th>SEMean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cadence</td>
<td>20</td>
<td>0.9245</td>
<td>0.9300</td>
<td>0.0809</td>
<td>0.0181</td>
</tr>
</tbody>
</table>

a. Calculate and interpret a 95% confidence interval for a population mean cadence. [5]

b. Calculate and interpret a 95% prediction interval for the cadence of a single individual randomly selected from this population [5]

c. Calculate an upper confidence bound with confidence level 90% for the population standard deviation of cadence. [5]
7. The financial manager of a large department store chain selected a random sample of 200 of its credit card customers and found that 136 had incurred an interest charge during the previous year because of an unpaid balance.

   a. Compute a 90% CI for the true proportion of credit card customers who incurred an interest charge during the previous year. [5]

   b. If the desired width of the 90% interval is 0.05, what sample size is necessary to ensure this? [5]

8. A manufacturer of College textbooks is interested in estimating the strength of the bindings produced by a particular binding machine. Strength can be measured by recording the force required to pull the pages from the binding. If this force is measured in pounds, how many books should be tested to estimate the average force required to break the binding within 0.1 lb with 95% confidence? Assume that σ is known to be 0.8. [5]

9. A 95% CI for population proportion of professional tennis players who earn more than $2 million a year is found to be [0.82, 0.88]. Given this information, find the sample size that was used to construct the CI. [5]
10. The superintendent of a large school district, having once had a course in probability and statistics, believes that the number of teachers absent on any given day has a Poisson distribution with parameter \( \lambda \). Use the accompanying data on absences for 52 days to derive a 95% CI for \( \lambda \). [Hint: The mean and variance of a Poisson variable both equal \( \lambda \), so \( Z = (\bar{X} - \lambda) / \sqrt{\lambda / n} \) has approximately a standard normal distribution.]

\[ \theta = \lambda, \hat{\theta} = \bar{X} \text{ and } \sigma_{\hat{\theta}} = \sqrt{\frac{\lambda}{n}} \text{ so } \sigma_{\hat{\theta}} = \sqrt{\frac{\bar{X}}{n}} \]  [10]

<table>
<thead>
<tr>
<th># Absences</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b. How large must \( n \) be if the width of the 99% interval for \( \mu \) is to be 1.0? [5]
11. A marketing company is planning a survey to estimate the proportion of high-school students read newspapers daily.

a. What sample size should be used if the company wants to give a 95% confidence interval which is guaranteed to be no more than 4 percentage points wide by assuming equal proportion for reading and not reading newspapers daily?[5]

b. If the company is certain that the true proportion of high school students who read a newspaper daily is less that 25%, what sample size should they use?[5]

12. The amount of lateral expansion (mils) was determined for a sample of $n = 9$ pulsed-power gas metal arc welds used in LNG ship containment tanks. The resulting sample standard deviation was $s = 2.80$ mils. Assuming normality, derive a 95% CI for $\sigma^2$ and for $\sigma$. [5].