1. Suppose $X_1, X_2, \ldots, X_n$ is a realization of a random sample from a continuous r.v. with the following density function:

$$f(x; \theta) = (\theta + 1)x^\theta \quad 0 \leq x \leq 1$$

Find the moment estimator of $\theta$. [5]

2. Suppose $X_1, X_2, \ldots, X_n$ is a realization of a random sample from a continuous r.v. with the following density function:

$$f_\theta(x) = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \quad \text{for} \quad x \geq 0$$

What is the likelihood function $L(\theta)$? What is the loglikelihood function $\ell(\theta)$? [5]
3. Suppose $X_1$, $X_2$, $X_3$ are i.i.d. Exp($\lambda$), and that we observe the realizations $X_1 = 1.0$, $X_2 = 2.0$, and $X_3 = 3.0$. What is the maximum likelihood estimate of $\Pr(X_1 > 2)$? [10]

4. A sample of 20 patients at a doctor's office reveals an average waiting time of 16 minutes, and a standard deviation of 5 minutes.

   (a) What is the point estimate of the average waiting time for all patients in this particular doctor's office? [2]

   (b) Calculate the (estimated) standard error of this point estimate, and explain what this number means. [5]

   (c) Explain why we cannot construct a confidence interval for the average waiting time for all patients using the Z distribution even if we could assume that the distribution of waiting time is normal. [2]
5. A sanitation department is interested in estimating the mean amount of garbage per bin for all bins in the city. In a random sample of 36 bins, the sample mean amount was 51.5 pounds and the population standard deviation was 4 pounds. Construct 99% confidence intervals for μ. [5]

6. As part of his class project, a Statistics student took a random sample of 50 College students and recorded how many hours a week they spent on the Internet. The sample reveals an average of 6.9 hrs. Calculate the 90% Confidence Interval for average Internet usage among college students. Assume that the standard deviation of Internet usage for college students is known to be $\sigma = 4.5$ hrs/week. [5]

7. What percentage of college students have made at least one online purchase in the last three months? To answer this question, a market researcher surveyed 200 college students. Of those surveyed, 76 said that they had made at least one online purchase. Calculate the corresponding point estimate and its standard error. [5]

8. A web based software company is interested in estimating the proportion of individuals who use the Firefox browser. In a sample of 200 of individuals, 31 users stated that they used Firefox. (a). Using this data, construct a 95% confidence interval for the proportion of all individuals that use Firefox. [5]
(b). What sample size would be required so that the width of the 95% confidence interval would be at most 0.08 units wide? [5]

9. The amount of lateral expansion (mils) was determined for a sample of \( n = 9 \) pulsed-power gas metal arc welds used in LNG ship containment tanks. The resulting sample standard deviation was \( s = 2.80 \) mils. Assuming normality, derive a 95% CI for \( \sigma^2 \). [5]

10. A sample of 14 joint specimens of a particular type gave a sample mean proportional limit stress of 8.50 MPa and a sample standard deviation of 0.80 MPa. Calculate and interpret a 95% prediction interval for the proportional limit stress of a single joint of this type. [5]