You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. No credit will be given for a problem if there is insufficient work/explanation.

1. A utility company offers a lifeline rate to any household whose electricity usage falls below 240 kWh during a particular month. Let A denote the event that a randomly selected household in a certain community does not exceed the lifeline usage during January, and let B be the analogous event for the month of July (A and B refer to the same household). Suppose $P(A) = 0.8$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$. What is the probability that the lifeline usage amount is exceeded in exactly one of the two months? [5]

2. A small manufacturing company will start operating a night shift. There are 20 machinists employed by the company. If a night crew consists of 3 machinists, and the machinists are ranked 1, 2, : : : , 20 in order of competence (1 being the best), how many of the crews would have at least 1 of the 10 best machinists? [5]

3. The Review editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60% of them are short, 30% are medium, and the other 10% are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80% are in Word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.
   a. What is the probability that the selected review was submitted in Word format? [5]
b. If the selected review was submitted in Word format, what are the posterior probabilities of it being short? [5]

4. Of the people who enter a blood bank to donate blood, 1 in 3 has type O+, and 1 in 20 has type O− blood. For the next three people entering the blood bank, let X denote the number with O+ blood. Find the probability distribution of X. [5]

5. I have a drawer that contains 10 white, 6 blue, and 4 brown socks.
   a. If I select socks at random with replacement, find the probability it takes me at least 3 tries to get a brown sock. [5]

   b. If I select 6 socks at random with replacement, find the probability that I get 3 white, 1 blue, and 2 brown socks in some order. [5]

6. A bowl contains 50 coins, ten Canadian dimes and 40 US dimes. A coin is selected at random, the type of coin is noted, and then the coin is set aside. The experiment is repeated 5 times. Let Y denote the number of Canadian dimes selected. What is P(Y=3)? [5]
7. A bowl contains 50 coins, ten Canadian dimes and 40 US dimes. A coin is selected at random, the type of coin is noted, and then the coin is returned to the bowl. The experiment is repeated until a Canadian dime is selected. Let $Y$ denote the number of times that the experiment is performed. What is $P(Y=3)$? [5]

8. Suppose that number of accidents at the Iowa power plant follows the Poisson process with the average rate of 0.40 accidents per week. Assume all weeks are independent of each other. Find the probability that exactly 4 accidents will occur in two months (8 weeks). [5]

9. In tests of stopping distances for automobiles, those automobiles traveling at 30 miles per hour before the brakes are applied tend to travel distances that appear to be uniformly distributed between two points $a$ and $b$. Suppose three automobiles are used in a test, find the probability that exactly one of the three travels past the midpoint between $a$ and $b$. [10]

10. Suppose that the wrapper of a certain candy bar lists its weight as 2.13 ounces. Suppose that the actual weights of these candy bars vary according to a normal distribution with mean $\mu = 2.2$ ounces and standard deviation $\sigma = 0.06$ ounces. In a random sample of 5 candy bars, what is the probability that at least one of the candy bars weighs less than the advertised weight? [5+5]
11. Suppose the survival time $X$ in weeks of a randomly selected male mouse exposed to 240 rads of gamma radiation has a gamma distribution with $\alpha = 8$ and $\beta = 15$. What is the probability that a mouse survives between 60 and 120 weeks? [5]

12. A random variable $Y$ has an exponential distribution with mean 3. An experimenter is interested in the function $h(Y) = e^{-Y}$
   
   a. Find $P(h(Y) \leq \frac{1}{8})$ [5]

   b. Find $E(h(Y))$ [5]
13. We know that the MGF of exponential distribution is $M_X(t) = \frac{\lambda}{\lambda - t}; \quad t < \lambda$. Using MGF show that the mean and variance are $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$. [10]

14. A health-food store stocks two different brands of a certain type of grain. Let $X =$ the amount (lb) of brand A on hand and $Y =$ the amount of brand B on hand. Suppose the joint pdf of $X$ and $Y$ is

$$f(x, y) = \begin{cases} kxy & x \geq 0, y \geq 0, \text{ and } 20 \leq x + y \leq 30 \\ 0 & \text{ otherwise} \end{cases}$$

a. Determine the value of $K$. [5]
b. Are X and Y independent? [5+5]

15. A candy company distributed boxes of chocolates with a mixture of creams, toffees, and nuts coated in both light and dark chocolate. For a randomly selected box, let X and Y, respectively, be the proportions of the light and dark chocolates that are creams and suppose that the joint density function is

\[ f(x, y) = \begin{cases} 
\frac{2}{5} (2x + 3y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\
0, & \text{elsewhere} 
\end{cases} \]

Find \( P((X, Y) \in A) \), where A is the region \( \{(x, y) \mid 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\} \). [5]
16. Let \( f_X(x) = 2x, \ 0 < x < 1 \). Given \( X = x \), suppose that \( Y \mid \sim U(0, x) \). Now find \( f_Y(y) \). Be sure to specify the range. [5]

17. Suppose that \( X \) and \( Y \) are both \( \text{Exp}(\lambda=1) \) random variables with \( \text{Cov}(X; Y) = 1/2 \). Find \( \text{Var}(3X + Y) \). [5]

18. Suppose \( X \) and \( Y \) are random variables such that \( \text{Var}(X + Y) = 15 \) and \( \text{Var}(X - Y) = 5 \). Find \( \text{Cov}(X, Y) \). [5]
19. Suppose $X_1; X_2; \ldots; X_{300}$ are i.i.d. Pois($\lambda=3$).

Further, define the sample mean $\bar{X} = \sum_{i=1}^{300} X_i$. Find $\Pr(2.9 \leq \bar{X} \leq 3.1)$. [10]

20. A student has a class that is supposed to end at 9.00 A.M and another class is supposed to begin at 9.19 A.M. Suppose the actual ending time of the 9 A.M class is normally distributed random variable $X_1$ with mean (μ) 9.02 and sd (σ) is 1.5 min and the starting time of the next class is also normally distributed random variable $X_2$ with mean 9.10 and sd is 1 min. Suppose also that the time necessary to get from one class to another class is also normally distributed random variable $X_3$ with mean 6 min and sd 1 min. What is the probability that the student makes it to the second class before the lecture starts? (Assume independence of $X_1$, $X_2$ and $X_3$). [10]