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MATH 321   Exam #1   100 Points                 Sep 21, 2012

You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. No credit will be given for a problem if there is insufficient work/explanation.

1. An urn has five balls, each of a different color. Ten balls are drawn from the urn, replacing the ball and mixing well after each draw. What is the probability that each color appears exactly twice? [5]

2. Two rabbits Flopsy and Peter might be found in a garden. At a certain time, the probability is 0.3 that Flopsy is in the garden, the probability is 0.8 that Peter is in the garden, and the probability is 0.2 that neither rabbit is in the garden. (The rabbits may not behave independently)
   (a) What is the probability that both Peter and Flopsy are in the garden? [5]

   (b) Given that Flopsy is in the garden, what is the probability that Peter is also in the garden? [5]

   (c) Let Y be the number of these two rabbits in the garden. Find E(Y). [5]
3. An airline has 2 pm, 8 pm, and 10 am flights from Toronto to Tokyo, Paris, and Seattle, respectively. Let A denote the event that “flight from Toronto to Tokyo is full”, and let B and C be analogous events for Paris and Seattle. Suppose that \( P(A) = 0.8 \), \( P(B) = 0.7 \) and \( P(C) = 0.6 \), and that the three events are independent. Find the probability that:
   (a) At least one flight is full. [5]
   (b) Only the Tokyo flight is full. [5]

4. In a genetics experiment involving crossing fruit flies, a male fly is equally likely to be genetic type A or B. If the fly is type A, then all offspring in a cross with a particular female will have red eyes. If the fly is type B, then each offspring in a cross with the same female is equally likely to have red eyes or not. Suppose that in the cross, there are six offspring, and all have red eyes. Given this additional information, what is the probability that the male fly is genetic type A? [10]

5. Individuals A and B begin to play a sequence of chess games. Let \( S = \{A \text{ wins a game}\} \), and suppose that outcomes of successive games are independent with \( P(S) = p \) and \( P(F) = 1-p \). They play until one of them wins ten games. Let \( X = \text{the number of games played.} \) Write the expression for \( P(X=x) \). [10]
6. You and a friend find a dime and a nickel. To decide who gets to keep them you flip the coins. If a coin comes up heads you win it; tails your friend wins it. Let S be the amount of money you win. What is the expected value of S? [5]

7. Suppose that a box contains 50 envelopes, 10 contain $8 and the others are empty. You choose 5 of the envelopes and will receive a reward of $3 plus the total amount of money contained in the 5 envelopes. Let Y denote the amount of money you receive. Find the mean, variance and standard deviation for Y. [5+5]

8. Given the 11 Scrabble letters making up the word “Shakespeare,” what is the probability that a monkey will misspell “Shakespeare” on each of the first four tries and then spell it correctly on the fifth try?[5]
9. Let us assume that in a certain city the probability that an earthquake occurs in each month is 1/100. Also assume that the event of an earthquake occurring (or not occurring) in a certain month is independent of whether it occurred in other months. An insurance company offers to insure a house against earthquakes for the value of 10,000 dollars. The cost of the insurance is 150 dollars a month. I.e. the house owner pays 150 dollars each month and if an earthquake occurs, the insurance pays 10,000 dollars to the owner, and at this stage the contract between the two parties terminates. What is the expected profit of the insurance company from insuring a single house? [5+5]

10. A reservation service employs five information operators who receive requests for information independently of one another, each according to a Poisson distribution with rate $\lambda=2$ per minute.
   1. What is the probability that during a given 1-min period, the operators receive no requests? [5]

   2. What is the probability that during a given 1-min period, exactly four of five operators receive no requests? [5]
11. If a (potential) basketball player arrives at the gym once every 4 minutes, how long will he have to wait for 3 more players to arrive (90% of times), so he can have a pick-up game? [10]