You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. **No credit** will be given for a problem if there is insufficient work/explanation.

1. Find the number of ways in which 4 boys and 4 girls can be seated in a row of 8 seats if they sit alternately. [2.5]

2. A student wishes to arrange 5 different history books, 3 different math books, and 4 different novels in a row on a shelf.
   a. In how many ways can she arrange the books? [2.5]

   b. In how many ways can she arrange the books if the books of each type must be together? [5]

3. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be no complete pair? [5]

4. Suppose that an ice cream parlor comes up with a clever promotion: customers roll two fair, six-sided dice and the price of a small cone (in cents) is the smaller number followed by the larger number, in cents. For example, if you roll a 5 and a 3, in either order, the price would be 35 cents. [Note that there are $6 \times 6 = 36$ equally likely outcomes in the sample space.]
   a. Determine the probability that the price would exceed 50 cents. [5]

   b. Determine the probability that the price is an odd number [5]
5. A customer has approached a bank for a loan. Without further information, the bank believes there is a 4% chance that the customer will default on the loan. The bank can run a credit check on the customer. The check will yield either a favorable or an unfavorable report. From past experience, the bank believes that $P(\text{favorable report being received } | \text{ customer will default}) = 0.03$ and $P(\text{favorable report being received } | \text{ customer will not default}) = 0.99$. What is the probability that a favorable report will be received? [5]

6. A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue operate in the city. Suppose you are told the following:
   - 85% of the cabs in the city are Green, and the remaining 15% are Blue.
   - A witness identified the cab as Blue (but it was dark outside!).

   The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time. What is the probability that the cab involved in the hit-and-run was actually blue rather than Green knowing that this witness identified it as Blue?" [10]

7. A husband and wife make their decisions independently of each other, and then they compare their decisions. If they agree, the decision is made; if they do not agree, then further consideration is necessary before a decision is reached. If they each have a history of making the right decision 70% of the time, what is the probability that together they delay the decision for further study? [5]
8. Let $Y$ be a random variable with support $\{1, 2, 3, 4, 5\}$ whose probability function $p(y)$ is given in the table below. (One value is omitted.)

<table>
<thead>
<tr>
<th>$y$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(y)$</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Find $P(Y = 4)$. [1]  
b. Find $P(Y$ is odd). [2]  
c. Find $P(Y \leq 2 | Y$ is odd). [3]  
d. Find $E(Y)$. [3]  
e. Find $V(Y)$. [3]  
f. Find $E(2^Y)$ [3]

9. An urn contains 30% Blue balls and 70% Green balls. The “Better-Pick-A-Blue” game at Bing Ohio Casino consists of drawing a sample of 5 balls with replacement from this urn. Let $X = \#$ of Blue balls in the sample. A player wins $1.10$ for every blue ball in the sample and the player loses $0.70$ for every green ball in the sample. The payoff for a play of this game depends on the observed value of $X$, say, $x$. Find the expected value and variance of the payoff. [10]
10. A recent study showed that only 20% of the women who lived with their boyfriends eventually walked down the aisle with them.

   a. Suppose that you randomly select women independently from person to person, what is the probability that 3 women must be sampled before 2 women married their boyfriend?[5]

   b. What is the expected number of women that you have to sample before you find one who married her boyfriend?[5]

11. An urn contains 4 red balls and 8 blue balls. Two people, Amy and Bob, play a game where they take turns drawing out one ball at a time, replacing and mixing the balls after each draw. Amy draws the first ball. Assume that for each draw, each ball in the urn is equally likely to be selected. **The winner is the person who draws the second red ball** (even if the other person drew the first red ball). [5+5+5=15]

   a. What is the probability that Bob wins the game on his fourth draw?

   b. What is the probability that the second red ball is drawn exactly six draws after the first red ball is drawn?
c. what is the probability that Amy is the person who draws the first red ball?

12. Suppose 6 criminals commit a violent crime, and are apprehended by the police. The police take 42 additional innocent people, and put all 48 in a line-up for a witness to select the 6 criminals. Unfortunately for the police, the witness is a pathological liar, and decides to randomly identify 6 from the lineup, none of the linees being shown any preference. What is the probability that the witness picks out exactly 2 of the criminals?[5]