MATH 321 Probability and Statistics I
Fall 2010
Mid Term Exam 1

NAME ________________________________ SECTION ______

Be sure to show all your work; your partial credit might depend on it. No credit will be given without supporting work.
The exam is closed book and closed notes. You are allowed to use a calculator and one 8½" x 11" sheet with notes on it.

1. What is the probability that the position in which the consonants appear remain unchanged when the letters of the word "Math" are re-arranged? [5]

\[ c, v, c, c \]

\[ \text{perm} = \frac{3 \cdot 1 \cdot 2 \cdot 1}{4!} \]

2. A man can hit a target once in 4 shots. If he fires 4 shots in succession, what is the probability that he will hit his target? [5]

\[ p(NH) = \frac{3}{4} \]

\[ p(H) = 1 - p(NH) = 1 - \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) \]

\[ = 1 - 0.3164 \approx 0.6836 \]

3. In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together? [5]

Arrange 4 things - 4! ways.

\[ \text{arrange ABACUS} \]

\[ 24 \text{ ways} \]

Since there are 2 A's, \[ \frac{3!}{2!} = 3 \]

\[ \frac{4! \cdot 3!}{2!} = 3 \times 24 = 72 \text{ ways.} \]
4. At Luther College, 25% of all students are enrolled in an introductory level statistics class, and 20% of all students are seniors. It is also known that 15% of the seniors are enrolled in an introductory level statistics class.

a) *Initech* is offering an internship to a Luther College student who is either a senior or is enrolled in an introductory level statistics class, or both. What proportion of Luther College students are eligible for this internship? [5]

\[ P(ST) = 0.25, \quad P(SR) = 0.20, \quad P(ST \mid SR) = 0.15 \]

\[ P(ST \cap SR) = P(ST \mid SR) \cdot P(SR) = (0.15)(0.20) = 0.03 \]

\[ P(ST \cup SR) = P(ST) + P(SR) - P(ST \cap SR) \]

\[ = 0.25 + 0.20 - 0.03 \]

\[ = 0.42 \]

b) What proportion of students enrolled in an introductory level statistics class at Luther College are seniors? [5]

\[ P(SR \mid ST) = \frac{P(SR \cap ST)}{P(ST)} = \frac{0.03}{0.25} = 0.12 \]

c) Are events \{a randomly selected student is enrolled in an introductory level statistics class\} and \{ a randomly selected student is a senior\} independent?

*Justify your answer. No credit will be given without proper justification.* [5]

\[ P(SR \cap ST) = 0.03 \]

\[ P(SR) \cdot P(ST) = 0.20 \cdot 0.25 = 0.05 \]

Since \[ P(SR \cap ST) \neq P(SR) \cdot P(ST) \]

SR and ST are not independent

(OR)

\[ P(ST \mid SR) = 0.15 \quad \text{and} \quad P(ST) = 0.25 \]

Since \[ P(ST \mid SR) \neq P(ST) \]
5. On any given day, the parents in a certain family remember to feed the family dog with probability 0.80. The children remember to feed the dog with probability 0.55. Assume independence.

a) Find the probability that on a given day either the parents or the children (or both) would remember to feed the dog. [5]

\[
P(\text{Par or Chi}) = P(\text{Par}) + P(\text{Chi}) - P(\text{Par} \cap \text{Chi})
\]

\[
= P(\text{Par}) + P(\text{Chi}) - P(\text{Par}) \times P(\text{Chi})
\]

\[
= 0.80 + 0.55 - (0.80)(0.55) = 0.91
\]

b) Find the probability that the dog would get fed exactly once on a given day. [5]

\[
P(\text{Par only}) + P(\text{Chi only})
\]

\[
= (0.80)(0.45) + (0.20)(0.55)
\]

\[
= 0.47
\]

6. Alex works the night shift at a gas station on the edge of Champaign. One night he mixed 3 stale doughnuts in with 12 fresh doughnuts. That night 6 customers bought one doughnut each, selecting them at random. What is the probability that at least 2 stale doughnuts were sold? [5]

\[
P(\text{atleast 2 stale}) = P(2 \text{ stale}) + P(3 \text{ stale})
\]

\[
= \binom{3}{2}\binom{12}{4} + \binom{3}{3}\binom{12}{1}
\]

\[
= \frac{15}{6} + \frac{15}{6}
\]

\[
= \frac{3 \times 495}{5005} + \frac{5 \times 220}{5005}
\]

\[
= 0.2957 + 0.0440
\]

\[
= 0.3407
\]
7. Alex is a naughty student – he does not always study for his exams. There is only a 60% chance that he would study for an exam. If he does study for an exam, the probability that he would pass it is 0.70. However, if he does not study, there is an 80% chance he would fail. Suppose you find out that Alex failed an exam. What is the probability that he did study for it?

\[
P(S) = 0.60 \quad P(F/S) = 0.70 \quad P(F/S') = 0.80
\]

\[
P(S/F) = \frac{P(SN|F)}{P(F)} = \frac{0.18}{0.5} = 0.36
\]

Using Bayes' theorem:

\[
P(S/F) = \frac{P(SN|F) \cdot P(S)}{P(F/S) \cdot P(S) + P(F/S') \cdot P(S')}
\]

\[
= \frac{(0.6)(0.30)}{(0.42)(0.30)+(0.08)(0.80)} = \frac{0.18}{0.50} = 0.36
\]

8. A consultant charges a company a retainer of $100 a week and $350 a day for services. The number of days per week the consultant works for this company is the random variable \( X \) with the distribution

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>0.25</td>
<td>0.25</td>
<td>0.20</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The weekly cost of this consultant to the company is then \( 350X + 100 \) (dollars).

a) Find the expected weekly cost of this consultant to the company. [10]

\[
E(X) = 1.85
\]

\[
E(350X + 100) = 350E(X) + 100
\]

\[
= (350)(1.85) + 100
\]

\[
= 747.5
\]

b) Find the standard deviation of this weekly cost. [10]

\[
\text{sd}(X) = \sqrt{\text{var}(X)} = \sqrt{\text{var}(350X + 100)}
\]

\[
= 350^2 \cdot \text{var}(X) = (350)^2 \cdot 2.6275
\]

\[
= 567.3348
\]